AP Calculus AB   Final Exam Review #2

For each problem, use implicit differentiation to find $\dfrac{dy}{dx}$ in terms of x and y.

1) $x^3 + 5y^2 = 5$

2) $3 = 4x^2 + 3xy^2$

For each problem, use implicit differentiation to find $\dfrac{dy}{dx}$ at the given point.

3) $5x^3 + 5 = \ln y^2$ at $(-1, 1)$

4) $2x^3 = -xy + 1$ at $(1, -1)$

For each problem, use implicit differentiation to find $\dfrac{d^2y}{dx^2}$ in terms of x and y.

5) $2x^3 - y^2 = 2$

For each problem, use implicit differentiation to find $\dfrac{d^2y}{dx^2}$ at the given point.

6) $2 = 2x^3 + 2y^2$ at $(-2, 3)$

For each problem, find the equation of the line tangent to the function at the given point. Leave answers in point-slope form.

7) $y = -x^3 + x^2 - 1$ at $(2, -5)$

8) $y = \dfrac{x^2}{2x - 2}$ at $\left(-3, -\dfrac{9}{8}\right)$

9) $y = -(-x + 2)^{\frac{1}{2}}$ at $(-2, -2)$

10) $y = \ln(x + 2)$ at $(4, \ln 6)$

For each problem, find the equation of the line normal to the function at the given point. If the normal line is a vertical line, indicate so. Otherwise, your answer should be in point-slope form.

11) $y = -x^3 + 4x^2 - 4$ at $(2, 4)$

12) $y = 2x^2 - 16x + 34$ at $(3, 4)$

13) $y = e^{-x-2}$ at $\left(-1, \dfrac{1}{e}\right)$

14) $y = -\dfrac{x^2}{2} + 2x - 2$ at $\left(1, -\dfrac{1}{2}\right)$

For each problem, find the points where the tangent line to the function is horizontal.

15) $y = -x^3 + 2x^2 + 3$

16) $y = 2x^2 + 4x - 4$

17) $y = \dfrac{x^2}{5x + 3}$

18) $y = -2x^2 - 4x - 2$

For each problem, find a tangent line approximation of the given quantity.

19) $\sqrt[3]{64.1}$

20) 2.97^4

21) $\sqrt{9.2}$

22) $\cos 881°$

For each problem, find the average rate of change of the function over the given interval.

23) $y = -\dfrac{1}{x + 1}$; $[1, 4]$

24) $y = \dfrac{1}{x}$; $[1, 3]$

25) $y = x^2 - 2x + 2$; $[1, 3]$

26) $y = x^2 - 2$; $[-3, -2]$

Answers to

1) $\dfrac{dy}{dx} = -\dfrac{3x^2}{10y}$

2) $\dfrac{dy}{dx} = \dfrac{-8x - 3y^2}{6xy}$

3) $\left.\dfrac{dy}{dx}\right|_{\substack{x=-1 \\ y=1}} = \dfrac{15}{2}$

4) $\left.\dfrac{dy}{dx}\right|_{\substack{x=1 \\ y=-1}} = -5$

5) $\dfrac{d^2y}{dx^2} = \dfrac{6xy^2 - 9x^4}{y^3}$

6) $\left.\dfrac{d^2y}{dx^2}\right|_{\substack{x=-2 \\ y=3}} = \dfrac{2}{3}$

7) $y = -8x + 11$

8) $y = \dfrac{15}{32}x + \dfrac{9}{32}$

9) $y = \dfrac{1}{4}x - \dfrac{3}{2}$

10) $y = \dfrac{1}{6}x + \dfrac{3\ln 6 - 2}{3}$

11) $y = -\dfrac{1}{4}x + \dfrac{9}{2}$

12) $y = \dfrac{1}{4}x + \dfrac{13}{4}$

13) $y = ex + \dfrac{e^2 + 1}{e}$

14) $y = -x + \dfrac{1}{2}$

15) $(0, 3), \left(\dfrac{4}{3}, \dfrac{113}{27}\right)$

16) $(-1, -6)$

17) $\left(-\dfrac{6}{5}, -\dfrac{12}{25}\right), (0, 0)$

18) $(-1, 0)$

19) $f(x) = \sqrt[3]{x},\ f'(x) = \dfrac{1}{3x^{\frac{2}{3}}}$

$x_0 = 64,\ \Delta x = 0.1$

$f(x_0 + \Delta x) \approx f(x_0) + f'(x_0)\Delta x = \dfrac{1921}{480} \approx 4.0021$

20) $f(x) = x^4,\ f'(x) = 4x^3$

$x_0 = 3,\ \Delta x = -0.03$

$f(x_0 + \Delta x) \approx f(x_0) + f'(x_0)\Delta x = \dfrac{1944}{25} = 77.76$

21) $f(x) = \sqrt{x},\ f'(x) = \dfrac{1}{2x^{\frac{1}{2}}}$

$x_0 = 9,\ \Delta x = 0.2$

$f(x_0 + \Delta x) \approx f(x_0) + f'(x_0)\Delta x = \dfrac{91}{30} \approx 3.0333$

22) $f(x) = \cos x,\ f'(x) = -\sin x$

$x_0 = \dfrac{\pi}{2}$ radians, $\Delta x = -\dfrac{\pi}{90}$ radians

$f(x_0 + \Delta x) \approx f(x_0) + f'(x_0)\Delta x = \dfrac{\pi}{90} \approx 0.0349$

23) $\dfrac{1}{10}$

24) $-\dfrac{1}{3}$

25) 2

26) -5